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Analysis of Neutron Streaming in Beam Tubes by a Favourable Coupling of Analytical Solution and S_N-Approximation of Transport Theory*)

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Abstract

A new method has been developed to treat the particle streaming through enlarged and extremely dimensioned cavities. It uses the analytical solution of the integral transport equation for the cavity zones and the S_N-approximation in the surrounding materials. The method has been applied to study streaming through beam tubes for high energy particle transport. In this context a comparison between different methods for the determination of spatial flux distributions for different beam tube dimensions bas been made.

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1 Introduction

The calculation of neutron and photon streaming through enlarged cavities is difficult, if one considers the interactions with the surrounding materials. Use of S_N-method /1/ necessitates high angular discretisation to describe a correct streaming through the cavities. Low orders lead to flux distortions, called "ray-effect" /2/. Other methods (e. g. Monte Carlo, spherical harmonics or first collision source) combined with the S_N-method are more complicated.

The transport equation can be solved analytically in vacuum zones. This solution can be embedded in the S_N-method with boundary conditions for cavity surfaces /3/. At the surface elements the incoming and outgoing directional fluxes are approximated by a polynomial expansion and the solution for the cavity is calculated with this approximation. A program RATRAC (Radiation Transport in Cavities), based on the above mentioned principle, has been developed and coupled with the two-dimensional S_N-code DOT (version 4.2) /4/. The method is realized in r,z-geometry for cylindrical and annular cavities or ducts /5/.

The method has been applied to analyse the streaming effects in beam tubes of the planned SNQ facility and the results have been compared with those from other methods. To show the spatial distributions for varied beam tube diameters the source has been defined.

2 Theory

The solution of the integral transport equation in vacuum (between two points on the cavity surface) is

$$\Phi(\vec{r}_p,\vec{\Omega}) = \Phi(\vec{r}_q,\vec{\Omega}) \tag{1}$$

where $\vec{r}_p$ indicated the target point and $\vec{r}_q$ the source point. From eq. (1) the particle current between the target area element dS_p and the source area element dS_q could be derived as below

$$(\vec{\Omega}\cdot\vec{n}_p)\Phi(\vec{r}_p,\vec{\Omega})d\vec{\Omega}dS_p = (\vec{\Omega}\cdot\vec{n}_p)(\vec{\Omega}\cdot\vec{n}_q)\Phi(\vec{r}_q,\vec{\Omega})\frac{1}{R_{pq}^2}dS_q dS_p \tag{2}$$

R_{pq} is the distance between $\vec{r}_p$ and $\vec{r}_q$.

To get a solution of eq. (2) the angular fluxes are approximated by Legendre polynomials. The functions for the approximation are so chosen that they are orthogonal in the 4 direction quadrants of the S_N-method. For any quadrant the directional flux density is given by

$$\Phi(\vec{r},\vec{\Omega}) = \sum_{\ell}\sum_{n<\ell}\frac{2\ell+1}{\pi}f_\ell^n(\vec{r})Y_\ell^n(\vec{\Omega}) \tag{3}$$

where f_1^n represents the moments of the expansion and Y_1^n are associated Legendre functions of order 1, n. Following three orthogonal systems allow above mentioned expansion for cylindrical geometry:

1, n even; (4)

1, n odd surfaces in radial directions; (5)

and

1 odd, n even surfaces in axial directions. (6)

In order to formulate a general expression of the directional flux density at the target point $\vec{r}_p$ in quadrant k the approximation in eq. (3) will be applied to eq. (2). Multiplying eq. (2) with Y_l^n at the target point and integrating over the quadrant k gives

$$\int_{\vec{\Omega}\varepsilon Q^k} Y_{\ell_p}^{n_p}(\vec{\Omega})(\vec{\Omega}\cdot\vec{n}_p)\Phi^k(\vec{r}_p,\vec{\Omega})dS_p d\vec{\Omega} = \int_{S_q} Y_{\ell_p}^{n_p}(\vec{\Omega})(\vec{\Omega}\cdot\vec{n}_p)(\vec{\Omega}\cdot\vec{n}_q)\Phi^{k'}(\vec{r}_q,\vec{\Omega}) \frac{dS_q dS_p}{R^2_{pq}} \quad . \quad (7)$$

On the left hand side of eq. (7) $(\vec{\Omega}\cdot\vec{n})Y_l^n(\vec{\Omega})$ will be expressed by different recursion formulas for the different surfaces. Use of the orthogonal system (5) in radial direction and of (6) in axial direction results in a system of linear equations for the moments f_l^n at the target point $\vec{r}_p$.

$$A\cdot FP = RP \qquad (8)$$

The matrix A contains the constant coefficients of the recursion formulas, F represents a vector with the moments f_l^n and the vector RP gives the solution of the right hand side of eq. (7). The solution of RP can be obtained by using system (4) for the angular dependence of the incoming fluxes.

$$RP_\ell^{nk} = \sum_{G'}\sum_{\ell_q}\sum_{n_q<\ell_q}\sum_{k'} \frac{2\ell_q+1}{\pi} \int_{u_q} f_{\ell_q}^{n_q k'}(\vec{r}_q) \{2\int_{\psi_q} Y_{\ell_p}^{n_p}(\vec{\Omega}_p) Y_{\ell_q}^{n_q}(\vec{\Omega}_q)$$

$$(\vec{\Omega}\cdot\vec{n}_p)(\vec{\Omega}\cdot\vec{n}_q)\cdot \frac{r_q}{R^2_{pq}} d\psi\} du_q dS_p \qquad (9)$$

The sums represent the streaming from different surfaces G', out of quadrant k' for the flux approximation l_q, n_q into quadrant k at surface G. The variable u_q means a surface element where $f_{l_q}^{n_q}(\vec{r}_q)$ is constant and the azimuthal angle ψ gives the possible optical view from source surface to the target point.

The coupling of above mentioned solution with the S_N-method needs the formulation of boundary conditions. These are defined by

$$f_{\ell_q}^{n_q k'}(\vec{r}_q) = \sum_{m \in Q^{k'}} \Phi_m(\vec{r}_q)\, w_m \quad \frac{\int_{\Delta\Omega_m} Y_{\ell_q}^{n_q} d\vec{\Omega}}{\int_{\Delta\Omega_m} d\vec{\Omega}} = 4\pi \sum_{m \in Q^k} \Phi_m(\vec{r}_q)\, w_m\, Y_{\ell_q m}^{n_q} \qquad (10)$$

for the incoming fluxes. The outgoing fluxes are given by

$$\Phi_m^k(\vec{r}_p) = \sum_{\ell_p} \sum_{n<\ell_p} \frac{2\ell_p+1}{\pi} f_{\ell_p}^{n_p k}(\vec{r}_p) \frac{\int_{\Delta\Omega_m} Y_{\ell_p}^{n_p}(\vec{\Omega}) d\vec{\Omega}}{\int_{\Delta\Omega_m} d\vec{\Omega}} \quad . \qquad (11)$$

By this formulation of the analytical solution of the integral transport equation and its coupling with S_N-approximation the streaming coefficients can be defined which describe the particle transport in cavities completely and depend only on the geometrical structure of the cavities. These coefficients can be calculated before flux iterations and stored for several problems with the same geometry of cavities. During the flux iteration cycle the treatment of the transport through cavities is reduced to matrix operations, which could be efficiently programmed (vectorized).

For extremely anisotropic outstreaming from cavities additional coefficients have been derived, which couple the instreaming on a source area element (eq. (10)) directly to the S_N-discretization on the target area elements.

By realization of the method following attributes have been included, which support the different scattering properties of the surrounding materials, different geometrical structures and computation technics:

- several different cavities can be defined,

- variable expansion orders on surfaces and quadrants are allowed,

- extremely anisotropic outstreaming can be treated additonally,

- variable S_N-orders are possible,

- coupling of RATRAC and DOT is realized efficiently for reducing computation time.

3 Results

A two-dimensional model in r-z-geometry was set up for the beam tube investigations (Fig. 1). Two cases with beam tube diameters 10 and 20 cm and length 600 cm were considered. Both were surrounded by a homogeneous iron shield of 200 cm diameter and 600 cm length. Since realistic source distributions in space and energy were not available at present state of SNQ project, they were defined artificially. At a distance of 50 cm from the beam tube entrance a constant flat source from r = 0 to r = 10 cm was defined. For this analysis the angular dependence of the source was taken as cosine shaped.

The cross sections for the iron shield have been taken from the high energy library from Los Alamos /6/. The library includes energies from thermal up to 800 MeV. The sources are defined in the 1st. (700 - 800 MeV), in the 15th. (1,2 - 1,0 MeV) and in the 32nd. group (1,235 - 0,454 keV) with the source density 1 n/cm^2. The results of one-group calculations are transferable to multigroups taking care of the ingroup scattering components.

The radial flux densities at the end of the beam tube are shown in Fig. 2 for the source with the highest energy. Two S_N-calculations, with an usual S_8 set and a biased S_N(166) set based on S_{10} and with a fine discretisation in -z direction, have been carried out. A calculation with the coupled method in $S_8/Y_2/D$(S_8 in scattering materials, an order of 2 in the expansions (Y_2) and using the additional coefficients (D) for cavity treatment) has been made. The line of sight results, which consider only the direct streaming from source area to the end of the beam tube, are shown in the same figure. It can be seen, that the S_8 calculation underestimates the fluxes very much compared with the other results. Use of $S_N(166)$ leads to an underestimation at the centre of the beam tube, but it gives a correct streaming through the outer surface of the beam tube. The solution of the coupled method represents the beam tube streaming exactly according to the line of sight results. For the energy group 1 the self scattering cross section is only 4,8 % of the total cross section, so that the scattering component for the duct streaming is very low. (The library includes only the inelastic scattering for high energies.)

It can be seen that the axial distribution, at r = 10,25 cm (Fig. 3), with $S_N(166)$ leads to some ray effects at the end of the beam tube. The results of the coupled method, using a S_8 set in the material zones are different, because this S_N set is not able to reproduce the extreme anisotropic outstreaming from the beam tube. The particle currents are correct, but the angular distribution of the particles is not adequately represented by a S_8-angular discretisation. To get better results near the beam tube one has to use finer angular discretisation like $S_N(80)/Y_2/D$ in Fig. 3.

An axial distribution at 190 cm (Fig. 4) shows that the differences between $S_N(166)$ and S_8 are less than a factor of two except for the flux distortions for $S_N(166)$ on the source side of the shield.

For the same source group the beam tube diameter is reduced to 10 cm (Fig. 5). A section of 100 cm radius at the end of the beam tube shows the underestimation by the $S_N(166)$ solution by a factor of 80. In contrast to the coupled method a new S_N set must be defined for smaller diameters.

As a last case the source energy has been changed (Fig. 6). The scattering component at the end of the beam tube increases only slightly with lower energies. For the 15th. group it is about 2 % compared to the first group. For the 32nd. group it is about 4 %, which can not be seen due to the logarithmic scale of the curves.

The radial distribution is quite different from those of the higher energies. This is due to decreasing of the mean free path lengths although the self-scattering rate increases from Σ_{g-g}/Σ_T = 0,048 for 700 - 800 MeV to 0,929 for 1,235 - 0,454 keV.

These results show that the new coupled method is able to treat the neutron streaming through extreme dimensional cavities, like beam tubes, without ray-effect. There are no special S_N sets to define for different beam tube dimensions, because the geometry will be analysed automatically from DOT input for generating the streaming coefficients.

Calculation time is more economic than using adjusted S_N sets. The method allows the continuation of the outstreaming at the end of the beam tube also. With variable S_N orders surrounding the beam tube the streaming along the beam tube can be treated by a low order and the outstreaming at the top of the beam tube can be determined exactly by high or biased S_N orders.

4 Literature

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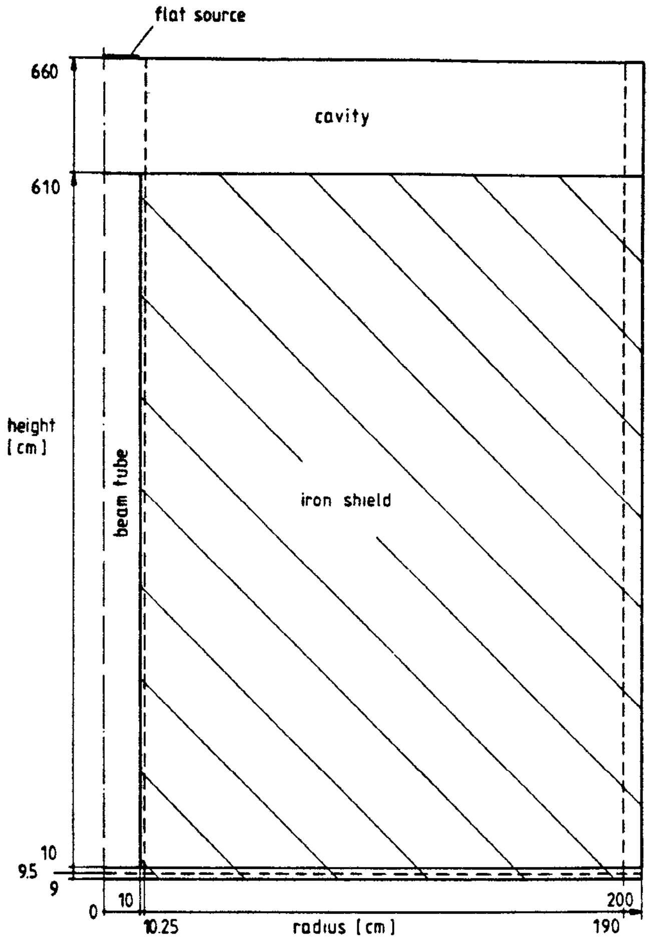
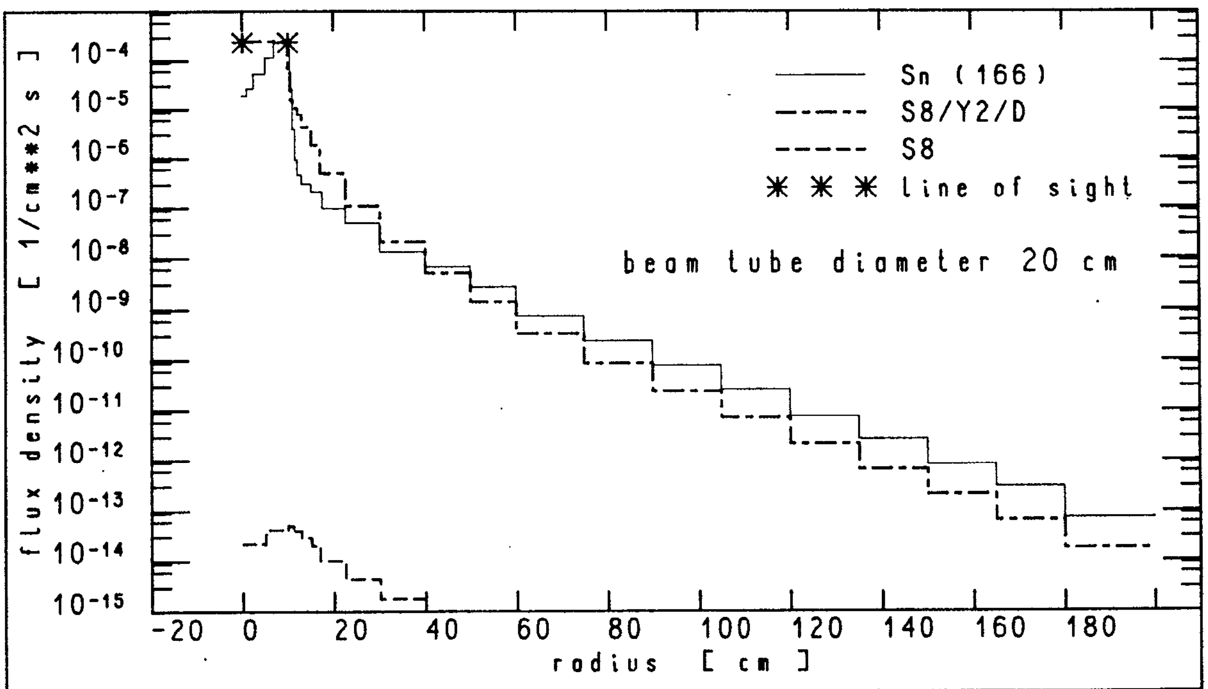
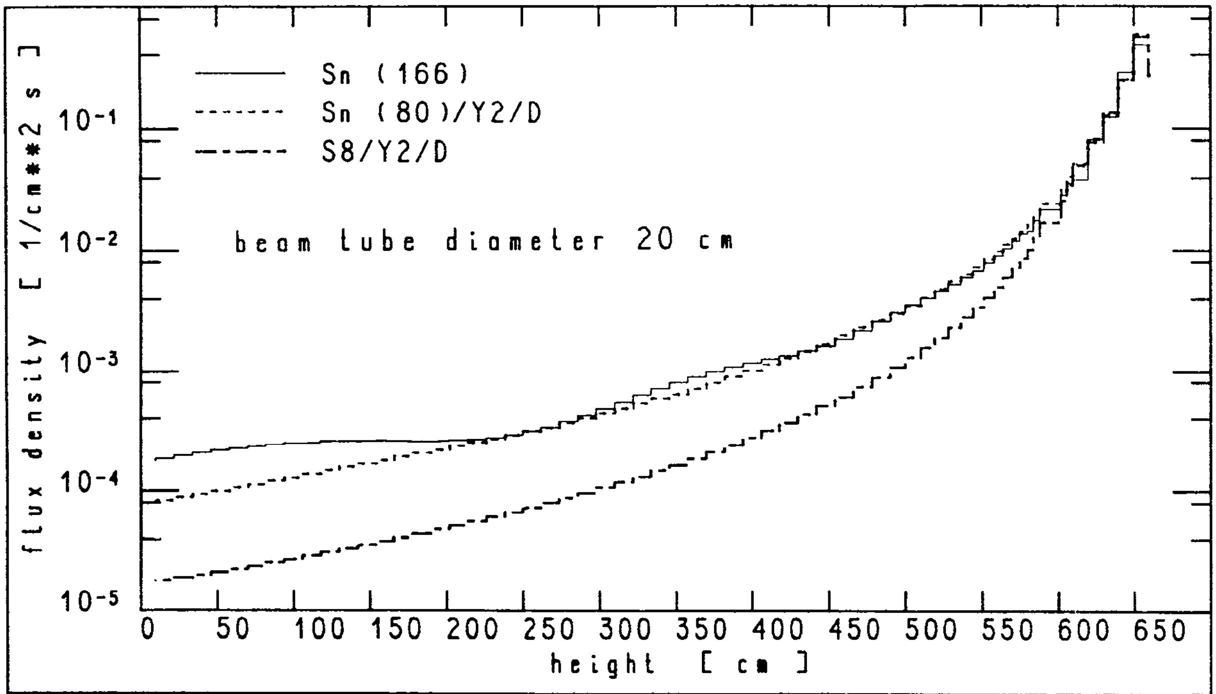


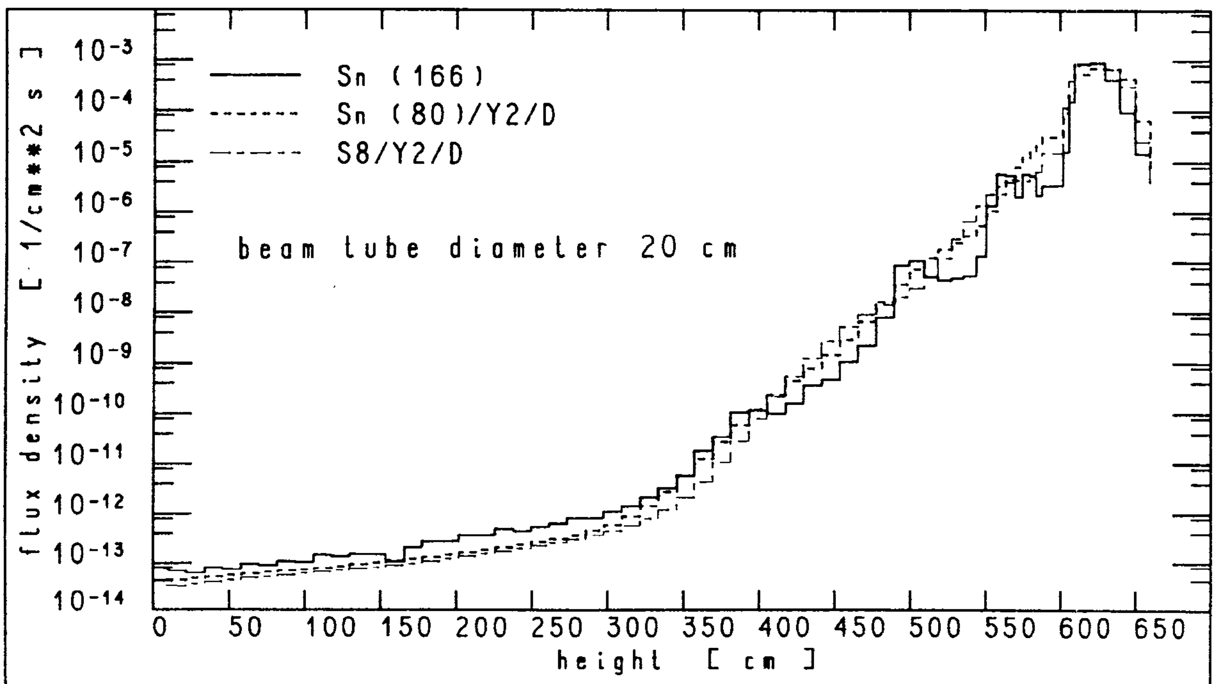
Fig 1: Geometry of the Model for the Beam Tube Investigations



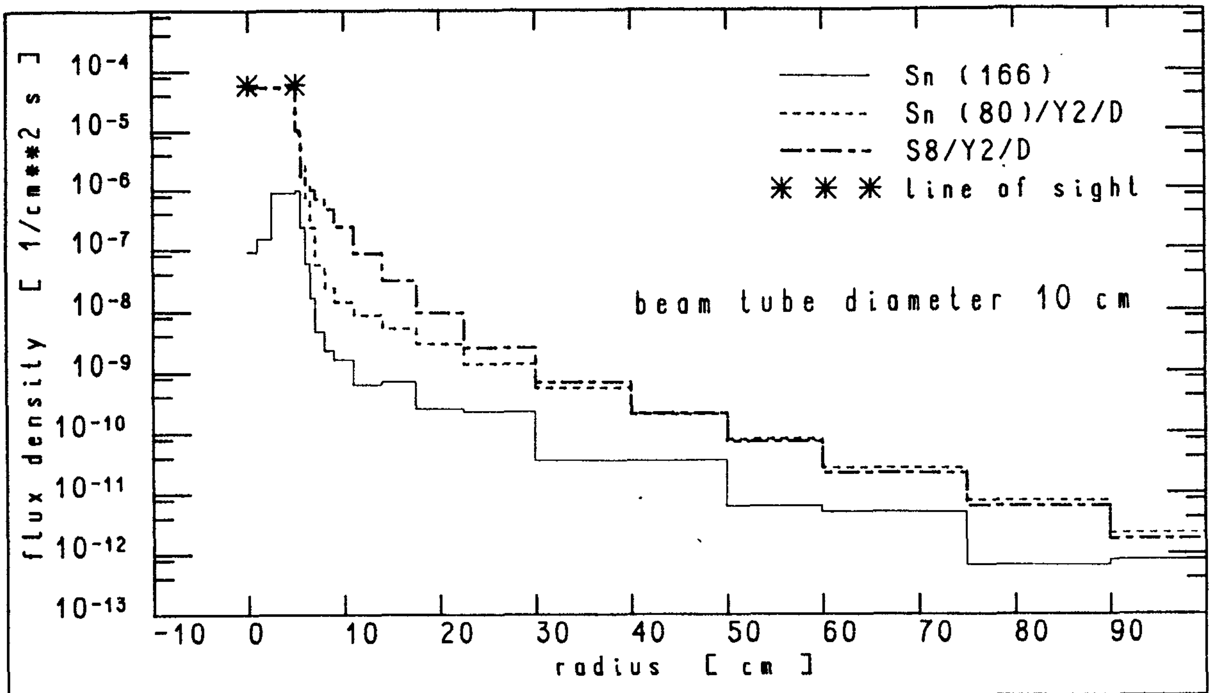
Radial flux distribution at the end of the beam tube (source 700-800 MeV)	IKE Stuttgart
	Fig. 2



Axial flux distribution along the beam tube at r=10.25 cm (source 700-800 MeV)	IKE Stuttgart Fig. 3



Axial flux distribution along the beam tube at r=190. cm (source 700-800 MeV)	IKE Stuttgart Fig. 4



Radial flux distribution at the end of the beam tube (source 700-800 MeV)

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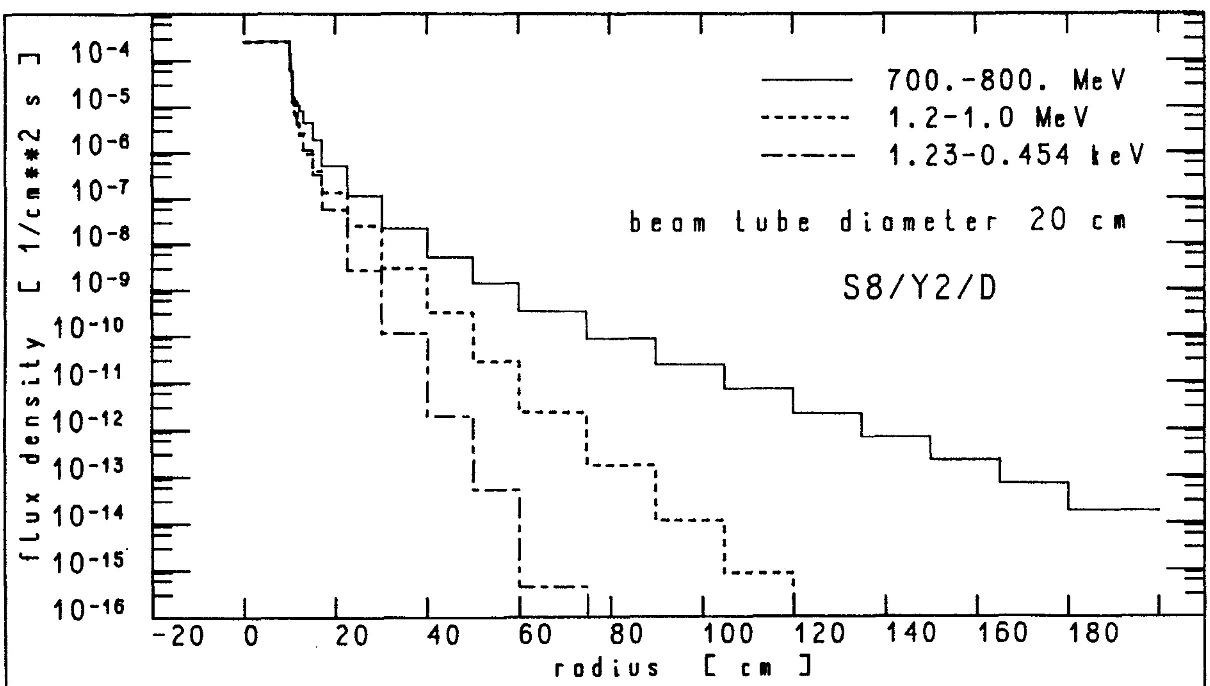
Fig. 5



Flux distribution at the end of the beam tube for different source energies

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Fig. 6