# On the use of acceptance diagrams to calculate the performance of multiple-section straight-sided neutron guide systems

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ABSTRACT: We describe a method to calculate the performance of multiple section systems of straight-sided guides and collimators. The approach is based on the concept of acceptance diagrams, previously described by J. M. Carpenter and D. F. R. Mildner [Nucl. Instrum. Meth. 196, 341 (1982)], which display the transverse spatial and angular coordinates of the neutrons in the system. For a given section of guide the construction of the exit diagram, from the entrance diagram, is shown to be accomplished using a shear transformation followed by translational and rotational operations applied to polygons representing respectively even and odd numbers of reflections within the section. The reflected neutron polygons are then truncated leaving only the neutrons that never strike a surface at an angle greater than the critical angle for total reflection.

## I. Introduction

In this paper we shall present a brief description of a generalization of the method of acceptance diagrams[1] which is used to study the behavior of neutron guide systems. The method may be applied to a wide variety of problems, including a parallel guide placed at a distance from a finite source[1], parallel[1], and converging[2] guides fed by an isotropic neutron source, and a converging guide following a long section of parallel guide[3]. Multiple-section systems may be handled, and angular and lateral displacements between sections are readily included. Open sections, which are equivalent to enclosed sections with non-reflecting walls, are simply treated as guide sections with zero critical angle for total reflection. The method is limited to one transverse dimension and to guides with straight sides, and the reflectivity of any given reflecting surface is assumed to be constant (not necessarily 100%) up to the critical angle. The source need not be uniformly illuminated, though calculations of intensity at the exit of the system are simplified if the source illumination is constant.

## II. Formulation

### IIa. Notation

The notation used to describe a typical guide section, and to characterize the trajectories of neutrons within the section, is illustrated in Fig. 1. The entrance and exit half-widths are W and W', the length is L, and the critical angle is $\theta$: if $\theta$ is zero we shall describe the section as a collimator. The neutron enters with spatial and angular coordinates y and $\beta$, and in the absence of any obstacle it strikes the exit plane (x = L) with coordinates y* and $\beta$*, where (in the small angle approximation)

$$y^* = y + L\beta, \text{ and} \tag{1a}$$

$$\beta^* = \beta. \tag{1b}$$

If the neutron reaches the exit plane, either without encountering the walls of the guide or else by reflection, it crosses with coordinates y' and $\beta'$.

The neutrons at a given stage within a system may be represented by an "acceptance diagram" which displays their transverse spatial and angular coordinates. A typical diagram consists of one or more polygons such that all points internal to each polygon represent accepted neutron.

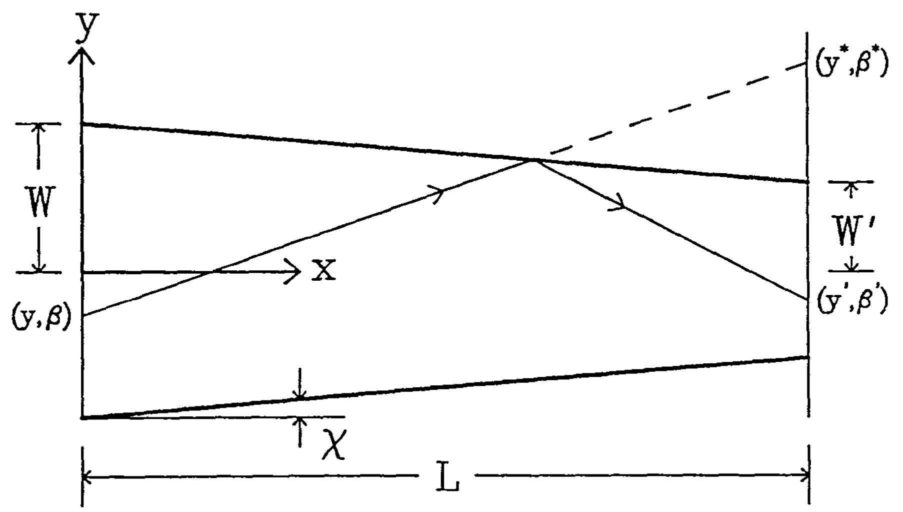


**Fig. 1** A typical section of a multiple section guide system. Angles are measured with respect to the positive x axis. In this example the taper angle $\chi$ is positive, and the trajectory of the neutron is such that $\beta > 0$, $\beta' < 0$, and k = 1.

## IIb. The exit acceptance diagram

Let us assume for the moment that all angles of reflection are permitted, and derive expressions for y' and β' in terms of y* and β*. As a first step we need to define and determine the reflection index k. The magnitude of k is the number of reflections suffered by the neutron, and its sign is the sign of the y-coordinate at the first reflection. For neutrons which reach the exit without reflection, the reflection index is zero. In Fig. 2 we show a converging guide together with some of its multiple images. Three possible neutron trajectories are shown, and we see that in general k is the integer κ which satisfies the inequality

$$(2\kappa - 1)\, W' \leq y^* \leq (2\kappa + 1)\, W' . \qquad (2)$$

For neutrons with $k = 0$ it is clear that $y' = y^*$ and $\beta' = \beta^*$, since no reflection occurs. If $k = 1$, $y' + y^* = 2W'$, and $\beta' + \beta^* = -2\chi$, where the taper angle $\chi$ (see Fig. 1) is given by

$$\chi = (W - W')/L. \qquad (3)$$

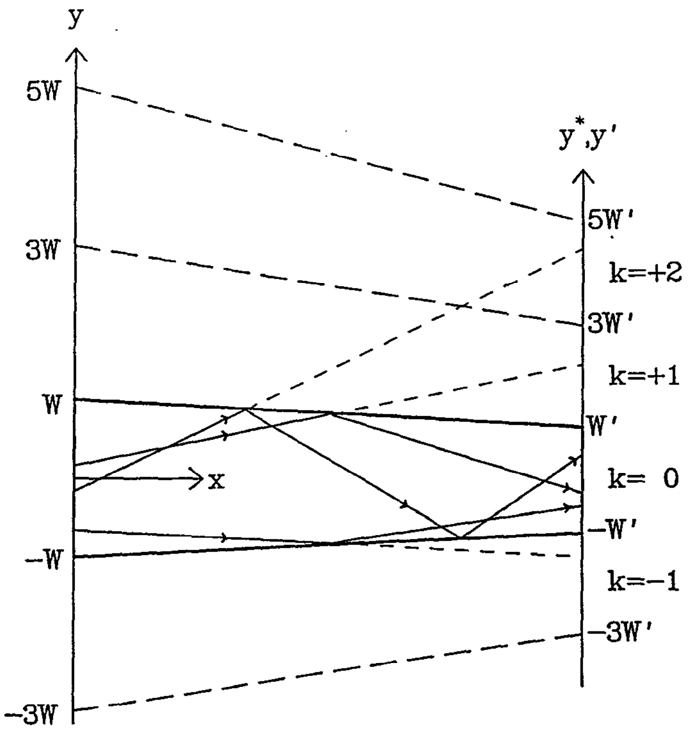


**Fig. 2** A converging guide and some of its images, shown as bold solid and dashed lines respectively. The trajectories of three neutrons are shown; in each case the dashed lines indicate trajectories in the absence of the guide.

Thus, for $k = 1$, $y' = -y^* + 2W'$ and $\beta' = -\beta^* - 2\chi$. For $k = 2$ we find that $y' = y^* - 4W'$ and $\beta' = \beta^* + 4\chi$. In general we obtain (cf Reference 2)

$$y' = y^* - 2\,kW';\ \beta' = \beta^* + 2\,k\chi \text{ for even } k, \tag{4}$$

$$y' = -y^* + 2\,kW';\ \beta' = -\beta^* - 2\,k\chi \text{ for odd } k, \tag{5}$$

We are now in a position to construct the exit acceptance diagram for a single section of guide, given the entrance acceptance diagram and assuming that all angles of reflection are allowed. The first step, illustrated in Fig. 3, is the shear transformation represented by Eqs. (1). Lines at $y^* = \pm(2\kappa - 1)W'$ (where $\kappa$ is an integer) are then added to the image acceptance diagram, Fig. 3(b), in order to classify neutrons according to their reflection index. The exit acceptance diagram is formed by taking the polygons for each value of k and applying the appropriate transformation, Eqs. (4) or (5): for even k the polygon is translated by addition of the vector $(-2kW', 2k\chi)$ whereas for odd k it is rotated through 180° about the point $(kW', -k\chi)$.

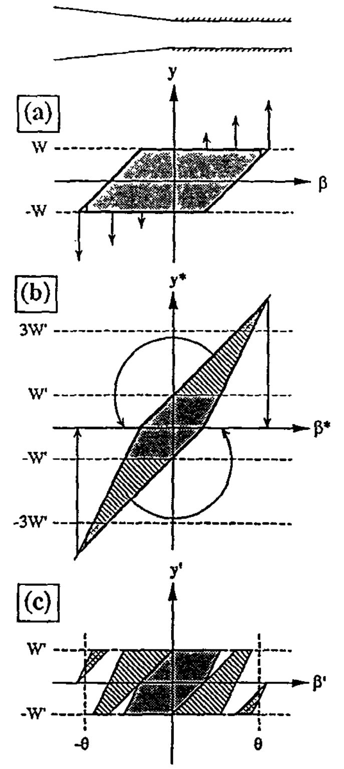


**Fig. 3** Acceptance diagrams for a parallel section of guide following a converging collimator. The collimator and guide have the same length L, and the collimator's entrance width is two times its exit width. The critical angle of the guide is 2.75 $\chi$, where $\chi$ is the taper angle of the collimator. The entrance diagram, (a), is a simple parallelogram. The image coordinate diagram, (b), is obtained by the shear transformation, Eqs. (1), and the exit diagram, (c), is obtained by a series of translational and rotational operations, Eqs. (4) and (5). The critical angle restriction, Eq. (6a), removes small portions from the polygons with $k = \pm 2$. The effect of the critical angle restriction is also evident in diagrams (a) and (b): the bold outlines were obtained by back-transformation of the accepted regions in diagram (c). Note that diagrams (a) and (c) are essentially identical to the acceptance diagrams in Fig. 6 of Reference 1.

## IIc. Critical angle conditions

If the section under consideration is a collimator, the only neutrons which reach the exit, are those with $k = 0$.

If the section is a converging guide, all neutrons with $k = 0$ reach the exit, and the additional neutrons (with nonzero k) which reach the exit are those that satisfy the critical angle condition on the final reflection[2, 3]

$$|\beta'| \leq \theta + \chi . \tag{6a}$$

This condition is applied to the exit diagram (excluding portions representing neutrons which reached the exit plane without reflection) after the transformations described by Eqs. (4) and (5) have been applied: it may or may not truncate polygons in the diagram.

If the section is a diverging guide, all neutrons with $k = 0$ reach the exit, and the additional neutrons which reach the exit are those that satisfy the following condition on the first reflection:

$$|\beta| \leq \theta - \chi . \tag{6b}$$

In this case the condition is applied to all parts of the image coordinate diagram with $|y^*| > W'$ (i.e., to all parts which represent reflected neutrons), before performing the transformations described by Eqs. (4) and (5): note that $\beta^* = \beta$.

In the case of a parallel guide conditions (6a) and (6b) are equivalent, since $|\beta| = |\beta'|$ and $\chi = 0$. An example of the application of condition 6(a) to a parallel guide section is illustrated in Fig. 3(c).

A second example of the procedure described in the previous paragraphs is shown in Fig. 4. The only difference between the arrangements shown in Figs. 3 and 4 is that in the latter case the guide converges to one half its original width.

## IId. Additional sections

Having determined the acceptance diagram at the exit of a given section we may proceed to the next section, assuming there is one. If the axis of the new section is aligned with the axis of the preceding section, the entrance diagram for the new section is identical to the exit diagram of the preceding section: an exception occurs if the new section is narrower than the preceding section. Angular and lateral misalignments of a section with respect to the preceding section may be readily handled by coordinate displacements. In the case of a spatial (sideways) displacement $v_i$ between sections i and i+1, the new and old y coordinates are related as follows:

$$y_{i+1} = y_i' - v_i, \tag{7}$$

whereas an angular displacement between successive sections, $\eta_i$, results in a change in the $\beta$ coordinates in analogous fashion:

$$\beta_{i+1} = \beta_i' - \eta_i. \tag{8}$$

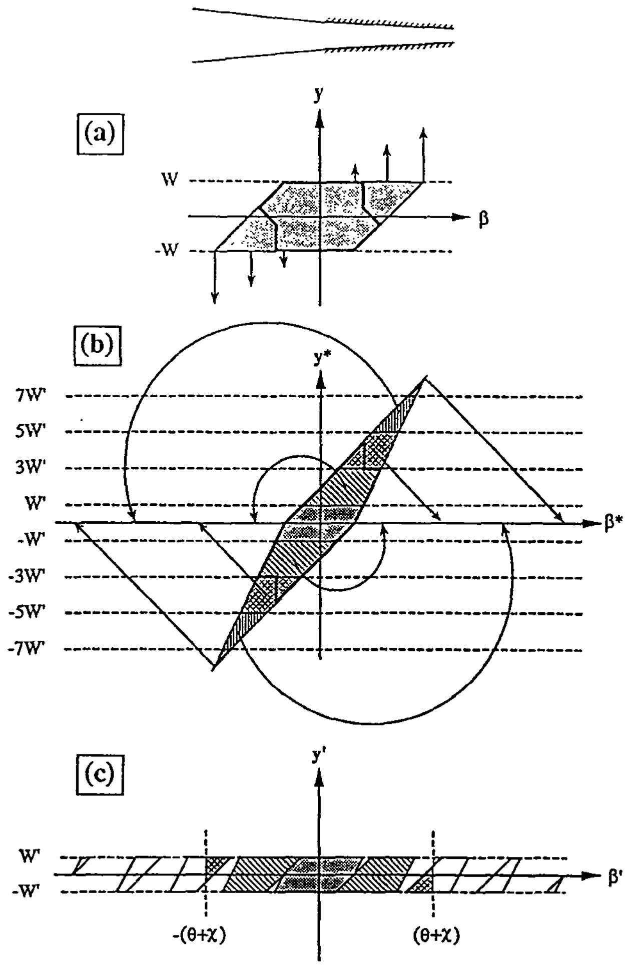


**Fig. 4** Acceptance diagrams for a system similar to the system depicted in Fig. 3, except that the guide converges so that its exit width is one-half its entrance width. Its critical angle is the same as that of the guide shown in Fig. 3. Diagrams (b) and (c) were obtained as described in the caption to Fig. 3. In this case the critical angle restriction, Eq. (6a), removes all neutrons with $|k| > 2$ and most of the neutrons with $|k| = 2$. The bold outlines in diagrams (a) and (b) were obtained by back-transformation of the accepted regions in diagram (c).

## IIe. The first section

The first section of a guide system is very often illuminated with neutrons traveling in all directions. In such cases the construction of the exit diagram for the first section differs somewhat from the procedure outline above, because the entrance diagram is bounded by the lines $y = \pm W$, but unbounded in $\beta$.

In the case of a converging section, the shear transformation, Eqs. (1), is performed first, followed by the transformations described by Eqs. (4) and (5). The only polygons which need be considered are those with $|k| \leq k_{max}$, where $k_{max}$ is the integer $\kappa$ which satisfies the inequality

$$2\,(\kappa - 1)\,W < L\,\theta \leq 2\,\kappa\,W. \tag{9a}$$

The result is one or more adjacent parallelograms, and some or all of the parallelograms representing reflected neutrons are then truncated according to the condition on the final reflection, eq. (6a).

In the case of a diverging section the shear transformation is again applied first, but polygons in the image coordinate diagram (outside the lines $|y^*| = W'$) are then truncated using the condition on the first reflection, Eq. (6b). The exit diagram transformations, Eqs. (4) and (5), are then performed. The only polygons which potentially contribute to the exit diagram are those with $|k| \leq k_{max}$, where $k_{max}$ is the integer $\kappa$ which satisfies the inequality

$$2\,(\kappa - 1)\,W' < L\,\theta \leq 2\,\kappa\,W'. \tag{9b}$$

## IIf. Intensity calculations

The relative intensities of neutrons at different planes within a guide system may be calculated using the relevant acceptance diagrams, as long as the reflectivity of the reflecting surfaces is constant for angles less than the critical angle. If the source illumination is uniform, the intensity at a given position is proportional to the sum of the areas of the polygons in the acceptance diagram. Thus the fraction of neutrons which reach the exit of the guide shown in Fig. 3, obtained from the ratio of the hatched areas in diagrams (c) and (a), is 0.992, whereas the same quantity for the guide shown in Fig. 4 is only 0.680, assuming the entrance to the guide is uniformly illuminated in both cases. In the absence of uniform illumination, the best method to determine which of the neutrons in the entrance diagram were able to reach the exit is to back-transform the accepted areas in the exit diagram, as illustrated in Figs. 3 and 4. Relative intensities may then be determined by integrating the source illumination $I(y,\beta)$ over appropriate regions in the entrance acceptance diagram.[4]

## III. Discussion

We have described a method to construct acceptance diagrams for systems of straight-sided guides and collimators. Relatively complicated arrangements can be treated in this way, and portions of complete systems may also be studied, since the illumination of the entrance to the first section need not be constant. The method may in principle be extended[4] to include situations where the critical angles of the two sides of a guide section are different, and to account for the neutrons which are transmitted by non-absorbing guide surfaces. It may be too that additional spectrometer components, such as filters and monochromators, can be treated using this type of approach. The alternative Monte Carlo method is more appropriate in situations where guide elements are continuously curved, and where surfaces do not have constant reflectivity.

## Acknowledgements

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## References

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2. D. F. R. Mildner, 1982, Nucl. Instrum. Meth. 200, 167.
3. I. S. Anderson, 1988, in Proceedings of 32nd Annual International Technical Symposium on Optical & Optoelectronic Applied Science and Engineering, San Diego, CA, 14-19 August 1988, paper 983-13.
4. J. R. D. Copley, paper in preparation.